2カラーQCDの低温高密度における相構造

PHASE STRUCTURE OF DENSE 2-COLOR QCD AT LOW TEMPERATURES

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based on arXiv:1910.07872

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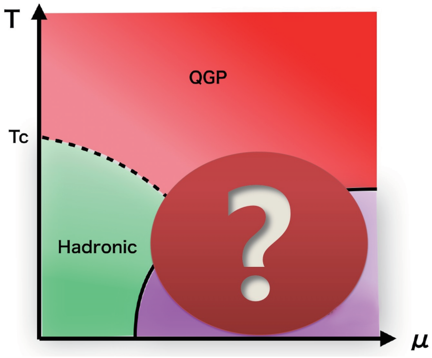
Outline

Introduction

Motivation

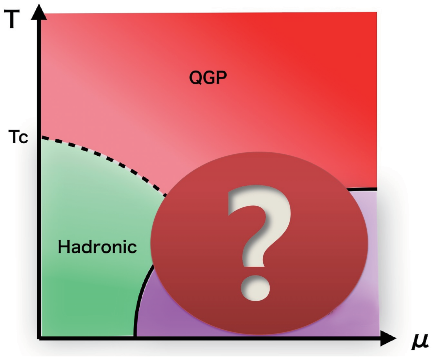
- QCD phase diagram:



- finite density regime is still less fully understood (inaccessible in lattice simulations due to sign problem)
- consider the SU(2) gauge theory (i.e. 2-color QCD)

Motivation

- QCD phase diagram:



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  (inaccessible in lattice simulations due to sign problem)
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Method

▸ sign

Why 2-color QCD?

- Similar nonperturbative properties

  e.g., color confinement, chiral symmetry breaking, ...

- Sign-problem free

  $\det[\Delta(\mu)] \in \mathbb{R}, \quad \det[\Delta^\dagger(\mu)\Delta(\mu)] > 0$ for even N_f

  $\Rightarrow$ MC calculations are feasible

But...

- Numerical instability $(\mu \gtrsim m_{\mathrm{PS}}/2)$ [Muroya-Nakamura-Nonaka 2001,2003] ▸ details

  $\Rightarrow$ introduce a diquark source $\quad (\det[\Delta(\mu)] \to \det[\Delta^\dagger(\mu)\Delta(\mu) + J^2]^{1/2})$

  [Kogut-Sinclair-Hands-Morrison 2001, Kogut-Toublan-Sinclair 2002, Alles-D'Elia-Lombardo 2006, etc]

  $$S_F = \bar\psi_1 \Delta(\mu)\psi_1 + \bar\psi_2\Delta(\mu)\psi_2 \underline{- J\bar\psi_1(C\gamma_5)\tau_2\bar\psi_2^T + \bar J\psi_2^T(C\gamma_5)\tau_2\psi_1}$$

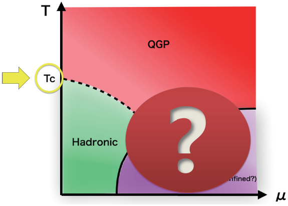
  $\Rightarrow$ take the vanishing limit of the source term $\quad (j \to 0, \; J = j\kappa)$

Here:

- Investigate the phase structure of QC$_2$D $\quad$ ($\rightarrow$ might shed light on real QCD)

Strategy

- determine T_c as a reference temperature (← chiral susceptibility peak)



(Our results: T_c lies at $\beta = 0.9, N_\tau = 10$)

- investigate the phase structure at two temperatures varying μ
  - ▷ $T = 0.45 T_c$ (density scan at cold regime)
  - ▷ $T = 0.89 T_c$ (density scan slightly below T_c)

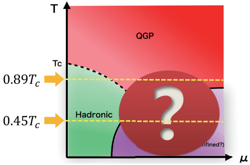
Simulation details

Lattice setup

details

- Lattice action: Iwasaki gauge action + 2-flavor Wilson fermion action
- Lattice size: $(N_s, N_\tau) = (16, 16)$ and $(32, 8)$
- Lattice parameters:
  - $(\beta, \kappa) = (0.800, 0.159)$ $[\to m_{\rm PS}/m_{\rm V} = 0.823,\ am_{\rm PS} = 0.623]$

    β: inverse gauge coupling, κ: hopping parameter, $m_{\rm PS}$: pseudoscalar meson mass, $m_{\rm V}$: vector meson mass
  - $a\mu \leq 1.0$ $(\mu/m_{\rm PS} \leq 1.6)$ (to avoid a lattice artifact)
- Two temperatures: temperatures $(T = \frac{1}{aN_\tau})$ corresponding to $N_\tau = 16, 8$ are found with lattice spacing a at $\beta = 0.8$ when $T/T_c = 1$ at $(\beta, N_\tau) = (0.9, 10)$.
  - $(\beta, N_\tau) = (0.8, 16) : T \simeq 0.45T_c$
  - $(\beta, N_\tau) = (0.8, 8) : T \simeq 0.89T_c$



Observables and Phases

▸ n_q

Observables:

- Polyakov loop: (approximate) order parameter of confined/deconfined phase

$$L = \frac{1}{N_s^3} \sum_{\vec{x}} \prod_{\tau} U_4(\vec{x},\tau) \;\rightarrow\; \langle L \rangle \sim e^{-F_q/T} \begin{cases} \langle L \rangle \sim 0 \;(F_q{=}\infty) : \text{confinement} \\ \langle L \rangle \neq 0 \;(F_q{=}0) : \text{deconfinement} \end{cases}$$

F_q: single quark free energy

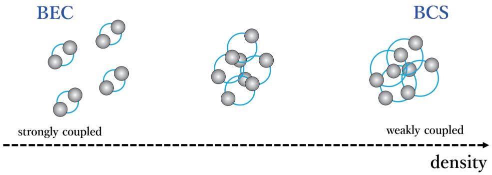
- diquark condensate: order parameter of superfluid phase

$$\langle qq \rangle = \frac{\kappa}{2} \langle \bar{\psi}_1 C\gamma_5 \tau_2 \bar{\psi}_2^T - \psi_1 C\gamma_5\tau_2\psi_2^T \rangle \;\rightarrow\; \begin{cases} \langle qq \rangle = 0 : \text{normal phase} \\ \langle qq \rangle \neq 0 : \text{superfluid phase} \end{cases}$$

- quark number density: criterion of BEC/BCS states

$$n_q < 0 : \text{BEC superfluid phase}, \quad n_q \sim n_q^{\text{tree}} : \text{BCS superfluid phase}$$

n_q^{tree}: quark number density described by a free field propagator at tree level



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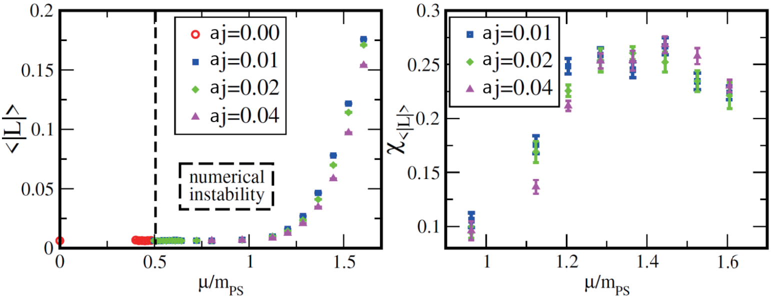
Our definition of phases:

	Hadronic	QGP	Superfluid	
			BEC	BCS
$\langle\lvert L\rvert\rangle$	$=0$	$\neq 0$	-	-
$\langle qq\rangle$	$=0$	$=0$	$\neq 0$	$\neq 0$
n_q	-	-	$n_q > 0$	$n_q/n_q^{\text{tree}}\sim 1$

Simulation results

phase structure at $T = 0.45T_c$

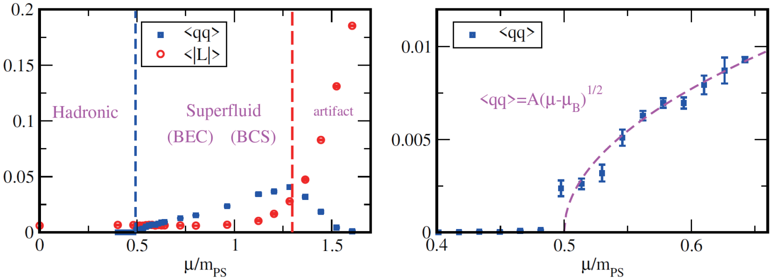
Polyakov loop



- simulations with j are feasible for $\mu/m_{\mathrm{PS}} \geq 0.50$
- nonzero value for $\mu/m_{\mathrm{PS}} \gtrsim 1.2$ (toward deconfinement transition?)
- susceptibility is marginally peaked at $\mu/m_{\mathrm{PS}} \sim 1.44$ $(a\mu \sim 0.90)$
  (but $a\mu$ is close to $1 \to$ lattice artifact??)

Diquark condensate

j-dep.

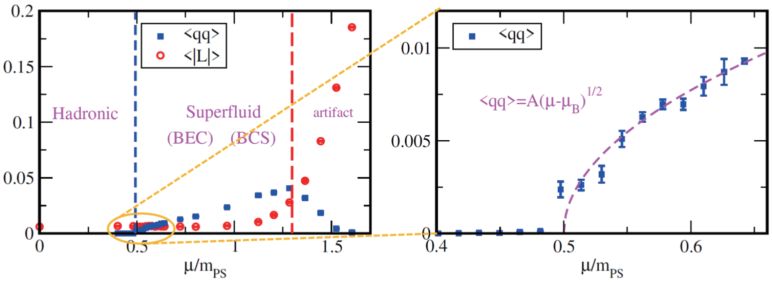


- transition from Hadronic to Superfluid transiton with increasing density
- transition point μ_B lies at $\mu/m_{\rm PS} \simeq 1/2$ (consistent with χPT prediction)
- condensates tend to decrease from $\mu/m_{\rm PS} = 1.28$ $(\mu \sim 0.80)$

  $\mu/m_{\rm PS} = 1.28 - 1.60$ $(\mu = 0.80 - 1.00)$ $\Rightarrow$ close to 1 lattice artifact?

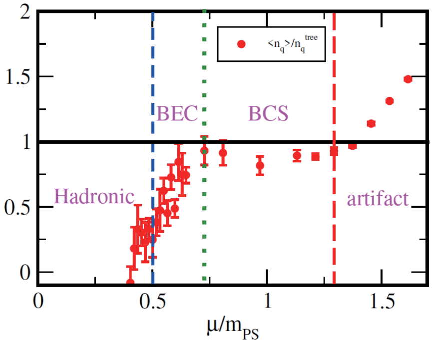
  (not only for staggered fermions [Kogut+ 2002, Braguta+ 2016] but for Wilson fermions)

Diquark condensate



- transition from Hadronic to Superfluid transiton with increasing density
- transition point μ_B lies at $\mu/m_{\rm PS} \simeq 1/2$ (consistent with χPT prediction)
- our data is consistent with theoretical curve on scaling low
  - ▷ scaling low around critical point μ_B: $\langle qq\rangle \propto (\mu-\mu_B)^{\beta_m}$
  - → give $\mu_B = m_{\rm PS}/2$ and $\beta_m = 0.50$ from mean-field predictions by χPT [Kogut *et al.* 2010]
  - → reasonable fitting ($\chi^2/\rm{d.o.f} = 1.31$)

Quark number density



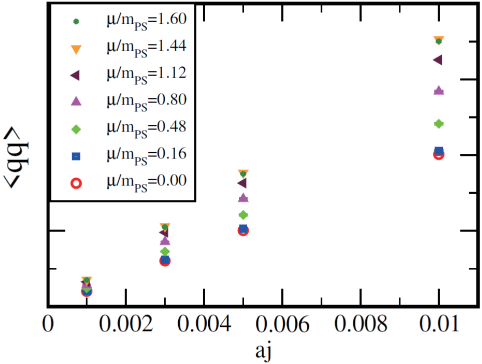
- weakly coupled BCS phase: $0.72 \lesssim \mu/m_{\text{PS}} \lesssim 1.28 \quad (0.45 \lesssim a\mu \lesssim 0.80)$
- strongly coupled BEC phase: $0.50 \lesssim \mu/m_{\text{PS}} \lesssim 1.72 \quad (0.31 \lesssim a\mu \lesssim 0.45)$
- nonzero n_q regime in Hadronic phase: $0.42 \lesssim \mu/m_{\text{PS}} \lesssim 0.50$ $(0.26 \lesssim a\mu \lesssim 0.31)$

  $\Rightarrow$ "Hadronic matter??" (in contrast to χPT prediction)

Simulation results

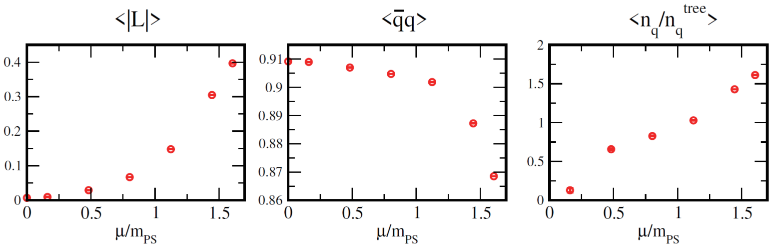
phase structure at $T = 0.89T_c$

Diquark condensate



- keep a zero value even for higher μ/m_{PS} regime in $j \to 0$ limit
- no superfluidity is observed in the μ-scan at $T = 0.89T_c$

Other observables

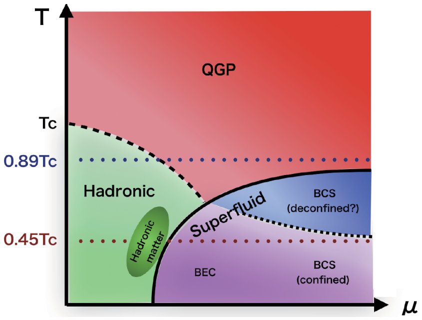


- Polyakov loop: tends to be deconfined with increasing density
- chiral condensate: tends to be chirally restored with increasing density
- quark number density: nonzero value early on around $\mu \approx 0.16$ ($\ll m_{\mathrm{PS}}/2$)

$\Rightarrow$ transition from Hadronic to QGP phase with increasing density

Summary

Summary



Phase structure at $T = 0.89T_c$

- system undergoes Hadronic-QGP transition
- there occurs no superfluid transition

$\Rightarrow$ Hadronic $\rightarrow$ QGP with increasing μ

Phase structure at $T = 0.45T_c$

- system undergoes Hadronic-Superfluid transition
- BEC and BCS states are identified

$\Rightarrow$ Hadronic $\rightarrow$ BEC $\rightarrow$ BCS with increasing μ

- nonzero n_q regime in Hadronic phase

$\Rightarrow$ found the "Hadronic matter"

- deconfined BCS superfluid transition has not observed this time...

but such a phase might exist in the intermediate-T and high-μ regime,
where a typical momentum of quarks p_F is larger than the size of the Fermi surface ($\sim \mu$).

Thank you for your kind attention!

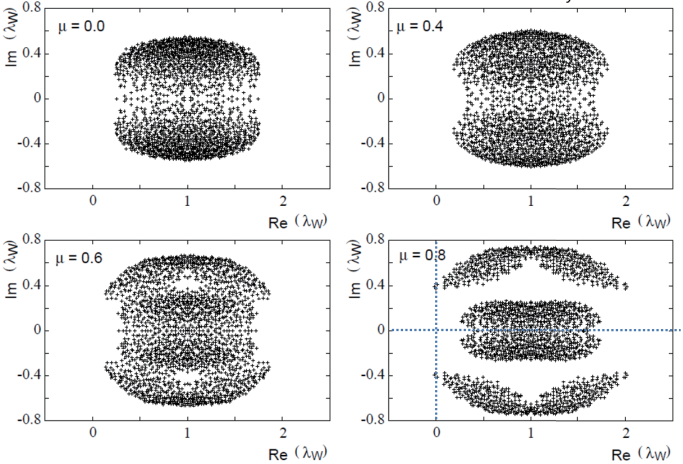
Backup Slides

Shell-bean structure

Return

Dirac eigenvalue distribution

Muroya et al. 2001



Lattice action

Return

- Iwasaki gauge action:

$$S_g = \beta \sum_x \left(c_0 \sum_{\substack{\mu<\nu \\ \mu,\nu=1}}^{4} W_{\mu\nu}^{1\times1}(x) + c_1 \sum_{\substack{\mu\neq\nu \\ \mu,\nu=1}}^{4} W_{\mu\nu}^{1\times2}(x) \right)$$

$$\beta = 4/g_0^2,\ c_1 = -0.331,\ c_0 = 1 - 8c_1$$

- 2-flavor Wilson fermion action:

$$S_F = \bar{\psi}_1 \Delta(\mu)\psi_1 + \bar{\psi}_2 \Delta(\mu)\psi_2 - J\bar{\psi}_1 (C\gamma_5)\tau_2 \bar{\psi}_2^T + \bar{J}\psi_2^T (C\gamma_5)\tau_2 \psi_1$$

- Wilson-Dirac operator:

$$\Delta(\mu)_{x,y} = \delta_{x,y} - \kappa \sum_{i=1}^{3} \left[(1-\gamma_i)U_{x,i}\delta_{x+\hat{i},y} + (1+\gamma_i)U_{y,i}^{\dagger}\delta_{x-\hat{i},y} \right]$$
$$-\kappa \left[e^{+\mu}(1-\gamma_4)U_{x,4}\delta_{x+\hat{4},y} + e^{-\mu}(1+\gamma_4)U_{y,4}^{\dagger}\delta_{x-\hat{4},y} \right]$$

Fermion matrix

Return

- fermion action with $\Psi = (\psi_1, C^{-1}\tau_2\bar{\psi}_2^T)^T$:

$$S_F = \bar{\Psi}\mathcal{M}\Psi$$

- extended fermion matrix (inverse Gorkov propagator):

$$\mathcal{M} = \begin{pmatrix} \Delta(\mu) & J\gamma_5 \\ -J\gamma_5 & \Delta(-\mu) \end{pmatrix}$$

- $\det[\mathcal{M}^\dagger\mathcal{M}]$ corresponds to 4-flavor fermion action

  $\rightarrow$ reduce to 2-flavor one:

$$\det[\mathcal{M}^\dagger\mathcal{M}]^{1/2} = \det[\Delta^\dagger(\mu)\Delta(\mu) + J^2]^{1/2}\det[\Delta^\dagger(-\mu)\Delta(-\mu) + J^2]^{1/2}$$

Lattice setup

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  - $(\beta, \kappa) = (0.800, 0.159)$ $[\to m_{\mathrm{PS}}/m_{\mathrm{V}} = 0.823(9),\ am_{\mathrm{PS}} = 0.623(3)]$
  - $a\mu \leq 1.0$ $(\mu/m_{\mathrm{PS}} \leq 1.6)$

Relation between β and a

reference scale: $t^2 \frac{t}{dt}(t^2\langle E\rangle)|_{t=w_0^2} = 0.3$

β	κ	w_0^2/a^2	$a/a_{\beta=0.9}$
0.8	0.1590	1.1519(23)	1.401(6)
0.9	0.1520	2.2616(149)	1

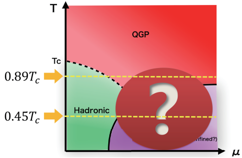
$T = 1/N_\tau a$ →

Temperature scale

β	N_τ	T/T_c
0.8	16	0.446(2)
0.9	10	1
0.8	8	0.892(4)

➡ $T \simeq 0.45T_c$

➡ $T \simeq 0.89T_c$



Observables and Phases

Return

Observables: order parameters characterizing each phase

- confined/deconfined phase: Polyakov loop

$$L = \frac{1}{N_s^3} \sum_{\vec{x}} \prod_{\tau} U_4(\vec{x}, \tau)$$

- superfluid phase: diquark condensate

$$\langle qq \rangle = \frac{\kappa}{2} \langle \bar{\psi}_1 C \gamma_5 \tau_2 \bar{\psi}_2^T - \psi_1 C \gamma_5 \tau_2 \psi_2^T \rangle$$

- BEC/BCS superfluid phase: quark number density

BEC: $a^3 n_q = \sum_i \kappa \langle \bar{\psi}_i(x)(\gamma_0 - \mathbb{I}_4) e^{\mu} U_4(x) \psi_i(x+\hat{4}) + \bar{\psi}_i(x)(\gamma_0 + \mathbb{I}_4) e^{-\mu} U_4^{\dagger}(x-\hat{4}) \psi_i(x-\hat{4})) \rangle$

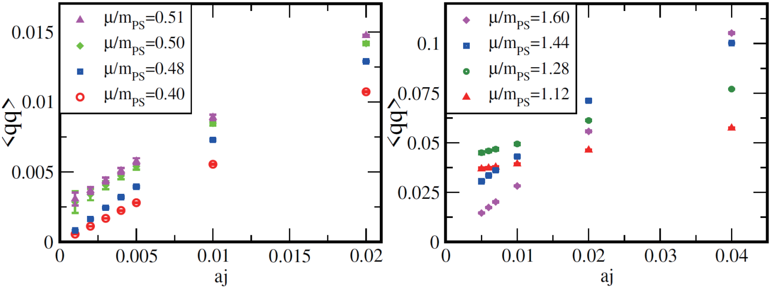
BCS: $n_q^{\text{tree}}(\mu) = \frac{4 N_c N_f}{N_s^3 N_\tau} \sum_k \frac{i \sin \tilde{k}_0 [\sum_i \cos k_i - \frac{1}{2\kappa}]}{[\frac{1}{2\kappa} - \sum_\nu \cos \tilde{k}_\nu]^2 + \sum_\nu \sin^2 \tilde{k}_\nu}$, $\quad \tilde{k}_0 = k_0 - i\mu = \frac{2\pi}{N_\tau}(n_0 + \frac{1}{2}) - i\mu$, $\tilde{k}_i = k_i = \frac{2\pi}{N_s} n_i, \ i = 1,2,3$

Our definition of phases:

	Hadronic	QGP	Superfluid	
			BEC	BCS
$\langle \lvert L \rvert \rangle$	$= 0$	$\neq 0$	-	-
$\langle qq \rangle$	$= 0$	$= 0$	$\neq 0$	$\neq 0$
n_q	-	-	$n_q > 0$	$n_q / n_q^{\text{tree}} \sim 1$

J-DEPENDENCE OF DIQUARK CONDENSATE

Return



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- condensates tend to decrease from $\mu/m_{\rm PS} = 1.28$ $(\mu \sim 0.80)$

  $\mu/m_{\rm PS} = 1.28 - 1.60$ $(\mu = 0.80 - 1.00)$ $\Rightarrow$ close to 1 lattice artifact?

  (not only for staggered fermions [Kogut+ 2002, Braguta+ 2016] but for Wilson fermions)

Sign problem

Return

- $Z = \int DU D\bar{q} Dq \exp(-S_G - S_F) = \int DU \det\Delta \exp(-S_G)$

  $\langle O \rangle = \frac{1}{Z} \int DU O \underline{\det\Delta \exp(-S_G)}$ probability

- $\Delta \;\rightarrow\; \Delta(\mu) = \not{D} + m + \mu\gamma^0$

  $h.c. \quad \Delta(\mu)^\dagger = \gamma^5 \Delta(-\mu) \gamma^5$

  $c.c. \quad [\det\Delta(\mu)]^* = \det\Delta(-\mu)$

- $\mu = 0 \;\rightarrow\; \det\Delta \in \mathbb{R}$

  $\mu \neq 0 \;\rightarrow\; \det\Delta \in \mathbb{C} \;\rightarrow$ MC infeasible (not positive probability)